

$$\widehat{GH}(z) = \frac{k(z-0.2)}{(z-1)(z+0.6)^2}$$

Find K For Stability

### 1 Bilinear Transformation

Ch. equation  $1 + \widehat{GH}(z) = 0$

$$(z-1)(z+0.6)^2 + k(z-0.2) = 0$$

$$(z-1)(z^2 + 1.2z + 0.36) + k(z-0.2) = 0$$

$$z^3 + 0.2z^2 + (k-0.84)z + (-0.2k-0.36) = 0$$

$$z = \frac{1+r}{1-r}$$

$$\left(\frac{1+r}{1-r}\right)^3 + 0.2\left(\frac{1+r}{1-r}\right)^2 + (k-0.84)\left(\frac{1+r}{1-r}\right) + (-0.2k-0.36) = 0$$

$$(1+r)^3 + 0.2(1+r)^2(1-r) + (k-0.84)(1+r)(1-r)^2 + (-0.2k-0.36)(1-r)^3 = 0$$

$$(1+r)^3 = r^3 + 3r^2 + 3r + 1$$

$$(1-r)^3 = -r^3 + 3r^2 - 3r + 1$$

$$(1-r)(1+r)^2 = -r^3 - r^2 + r + 1$$

$$(1+r)(1-r)^2 = r^3 - r^2 - r + 1$$

$$r^3 + 3r^2 + 3r + 1 - 0.2r^3 - 0.2r^2 + 0.2r + 0.2$$

$$+ kr^3 - kr^2 - kr - k - 0.84r^3 + 0.84r^2$$

$$+ 0.84r - 0.84 + 0.2kr^3 - 0.6kr^2$$

$$+ 0.6kr - 0.2k + 0.36r^3 - 1.08r^2$$

$$+ 1.08r - 0.36 = 0$$

$$(0.32 + 1.2)r^3 + (2.56 - 1.6k)r^2$$

$$+ (5.12 - 0.4k)r + 0.8k = 0$$

Routh Array

$r^3$	$(0.32 + 1.2k)$	$(5.12 - 0.4k)$
$r^2$	$(2.56 - 1.6k)$	$0.8k$
$r^1$	$\frac{(2.56 - 1.6k)(5.12 - 0.4k) - 0.8k(0.32 + 1.2k)}{(2.56 - 1.6k)}$	
$r^0$	$0.8k$	

For stability all first elements

must have the same sign

positive or negative

Sec 2

all positive

$$0.8k > 0 \rightarrow k > 0$$

and

$$0.32 + 1.2k > 0 \rightarrow k > -0.26$$

and

$$2.56 - 1.6k > 0 \rightarrow k < 1.6$$

and

$$\frac{(2.56 - 1.6k)(5.12 - 0.4k) - 0.8k(0.32 + 1.2k)}{2.56 - 1.6k} > 0$$

$$-(0.32k^2 + 9.576k + 13.1072) > 0$$

$$(k - 1.32)(k + 30.92) \neq 0$$

$$k - 1.32 > 0$$

$$k > 1.32$$

and

$$k + 30.92 \neq 0$$

$$k \neq -30.92$$

~~$k \neq 1.32$~~

OR

$$k - 1.32 < 0$$

$$k < 1.32$$

~~$k + 30.92 \neq 0$~~

$$k \neq -30.92$$

$$-30.92 < k < 1.32$$

$k$  for Stability

$$0 < k < 1.32$$

### 2 Jury method

Ch. equation

$$F(z) = z^3 + 0.2z^2 + (k-0.84)z + (-0.2k-0.36)$$

$$\textcircled{1} F(1) > 0$$

$$1 + 0.2 + k - 0.84 - 0.2k - 0.36 > 0$$

$$0.8k > 0$$

$$k > 0$$

$$\textcircled{2} F(-1) (-1)^n > 0$$

$$-1.2k + 0.32 > 0$$

$$k > -0.267$$

$$③ |z_0| < |z_n|$$

$$|+0.2K + 0.36| < 1$$

$$-1 < +0.2K + 0.36 < 1$$

$$-6.8 < K < 3.2$$

	$z^0$	$z^1$	$z^2$	$z^3$
1	$(-0.2K - 0.36)$	$(K - 0.84)$	0.2	1
2	1	0.2	$(K - 0.84)$	$(-0.2K - 0.36)$
3	$b_0$	$b_1$	$b_2$	

$$b_0 = \begin{vmatrix} -0.2K - 0.36 & 1 & \cancel{K - 0.84} \\ 1 & -0.2K - 0.36 \end{vmatrix}$$

$$b_0 = (0.2K + 0.36)^2 - 1$$

$$b_0 = 0.4K^2 + 0.144K - 0.8704$$

$$b_3 = \begin{vmatrix} -0.2K - 0.36 & K - 0.84 \\ 1 & 0.2 \end{vmatrix}$$

$$b_3 = (-0.2K - 0.36)0.2 - (K - 0.84)$$

$$b_3 = -1.04K + 0.768$$

$$④ |b_0| > |b_{n-1}|$$

$$|0.4K^2 + 0.144K - 0.8704| > |-1.04K + 0.768|$$

$$\left| \frac{0.4K^2 + 0.144K - 0.8704}{-1.04K + 0.768} \right| > 1$$

$$|M| > 1$$

$$\cancel{M} > 1 \text{ or } M < -1$$

$$④ M > 1$$

$$\frac{0.4K^2 + 0.144K - 0.8704}{-1.04K + 0.768} - 1 > 0$$

$$\frac{0.4K^2 + 0.144K - 0.8704 + 1.04K - 0.768}{-1.04K + 0.768} > 0$$

$$\frac{(K - 1.027)(K + 3.98)}{-1.04K + 0.768} > 0$$

$$i) K > 1.027 \text{ and } K > -3.98$$

$$\text{and } K < 0.738 \rightarrow \emptyset$$

$$ii) K > 1.027 \text{ and } K < -3.98$$

$$\text{and } K > 0.738 \rightarrow \emptyset$$

$$iii) K < 1.027 \text{ and } K > -3.98$$

$$\text{and } K \geq 0.738 \rightarrow 0.738 < K < 1.027$$

$$iv) K < 1.027 \text{ and } K < -3.98$$

$$\text{and } K \leq 0.738 \rightarrow K < -3.98$$

$$0.738 < K < 1.027$$

$$④ M < -1$$

$$\frac{0.4K^2 + 0.144K - 0.8704}{-1.04K + 0.768} + 1 < 0$$

$$\frac{(K - 2.11)(K - 0.12)}{1.04K - 0.768} > 0$$

$$i) K > 2.11 \text{ and } K > 0.12$$

$$\text{and } K > 0.738 \rightarrow K > 2.11$$

$$ii) K < 2.11 \text{ and } K < 0.12$$

$$\text{and } K > 0.738 \rightarrow \emptyset$$

$$iii) K < 2.11 \text{ and } K > 0.12$$

$$\text{and } K < 0.738 \rightarrow 0.12 < K < 0.738$$

$$iv) K > 2.11 \text{ and } K < 0.12$$

$$\text{and } K < 0.738 \rightarrow \emptyset$$

$$0.12 < K < 0.738 \text{ or } K > 2.11$$

$$④ 0.12 < K < 1.027 \text{ or } K > 2.11$$

for Stability

$$0.12 < K < 1.027 \text{ and } 2.11 < K < 3.2$$